1 informal proofs

proofs are where we can use what we have learned so far with propositional equivalence and rules of inference. in this chapter we cover basic fundamental proof techniques and strategies to prove mathematical theorems, in particular to those stated as implications.

1.1 direct proof

for a direct proof, we assume that the premise p is true and then show that the conclusion q must follow.

1.1.1 example

Theorem: if n is an odd integer, then n^3 is an odd integer.
Proof:
1. **assume p is true**: assume n is an odd integer. this means n can be written as $2k+1$ for some integer k.
2. **show q must be true**: substitute $(2k+1)$ for n in n^3:
   - $n^3 = (2k+1)^3 = 8k^3 + 12k^2 + 6k + 1$
3. **conclude**: after factoring the expression on the right, we get $n^3 = 2(4k^3 + 6k^2 + 3k) + 1$, and since k is an integer, $4k^3 + 6k^2 + 3k$ is just another big integer, and we can call it m. we can replace that in the factored equation, which gives us $n^3 = 2m + 1$ – the definition of an odd integer. ■

1.2 proof by contraposition

proof by contraposition is a type of indirect proof, where we prove the *contrapositive* of an implication, which is $\neg q \to \neg p$ – logically equivalent to $p \to q$.

1.2.1 example

Theorem: if n is an integer and n^2 is even, then n is even.
Proof: we can try doing a direct proof first:
- assume that n^2 is even, thus $n^2 = 2k$ for some integer k
- square root on both sides: $n = \sqrt{2k}$
- ...?

at this point, it is difficult to algebraically prove that n must be an even integer. instead, we use proof by contraposition, since the negation of the conclusion is easier to work with than the original hypothesis.

Proof:
1. **assume $\neg q$**: assume n is odd; thus $n = 2k$ for some integer k.

2. $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$
3. **show $\neg p$**: $= 2(2k^2 + 2k) + 1; m = (2k^2 + 2k); = 2m + 1$
4. $n^2 = 2m + 1$

thus, n^2 is odd, and we proved $\neg(\text{n is even}) \to (n^2 \ \text{ is even})$, we can conclude that n^2 is even $\to n$ is even. ■

1.3 proof by contradiction

1.4 which to choose