1 nested quantifiers

$$\forall x \, \exists y \, \forall z \, [(x + y) \cdot z = 0]$$

translation: for all x, there exists a y such that for all z, the result of x plus y, multiplied by z, is zero.

1.1 nested quantifier order

the order of the quantifiers matter! when reading from left to right, *later* quantifiers are within the scope of the *earlier* ones.

$$\forall x \, \exists y \, (x + y = 0) \quad ✓$$
$$\exists y \, \forall x \, (x + y = 0) \quad ×$$

1.2 translation

yes, we can (obiviously) still formalize sentences from english. let's see some examples.

1.2.1 example 1

take this statement:

every real number except zero has a multiplicative inverse.

we can rewrite that into a sentence that is easier for us to translate:

for **every** real number x, if $x \neq 0$,
then there **exists** a real number y such that $x \cdot y = 1$.

finally, we can formalize that sentence:

$$\forall x \, [(x \neq 0) \rightarrow \exists y \, [x \cdot y = 1]]$$

1.2.2 example 2

take this statement:

every student has at least one friend that is dating a steelers fan.

we first need to rewrite our statement again:

for **every** student x, then
there **exists** a friend of theirs who is dating a steelers fan.

Let:

- $S(x) \equiv$ "x is a student"
- $F(x, y) \equiv$ "x is friends with y"
- $D(x, y) \equiv$ "x and y are dating"
- $E(x) \equiv$ "x is a steelers fan"

$$\forall x \left[S(x) \to \exists y \left[F(x, y) \land D(y, z) \land E(z)\right]\right]$$