closures

the set of all regular languages are closed under three operations – meaning that the results after the operation will still be in the same set – **union**, **concatenation**, and **star**.

> **theorem**: the set of regular languages is closed under the regular operations.

union

if A and B are regular languages, then $A \cup B$ is also regular. this new regular language C recognizes strings that are either in A or in B.

we can construct a machine N where $L(N) = L(N_1) \cup L(N_2)$ from two machines N_1 and N_2 like this:

- draw the state diagram of N_1 on the top half
- draw the state diagram of N_2 on the bottom half
- add ϵ transitions from the new start state to the start states of N_1 and N_2 respectively.

it looks like this:

you might noticed the strange diagram. it might look complicated, but it is simply an abstraction; each rounded box, or "bubble", is a finite automaton; inside the box there are the all the states and transitions.

by abstracting away the detail in the automaton, it helps us to visualize the operations better.

concatenation

if A and B are regular languages, then $A \circ B$ (concatenation) is regular too. this new language recognizes strings that start with something in A and end with something from B.

to construct the concatenation, we

- draw A on the left and B on the right
- for each accept state in A, add an ϵ transition pointing to the start state of B.
- change all the accept states of A to regular non-accept state.

it looks like this:

star operation

if A is regular, $A*$ is also regular. the star operation recognizes ϵ or any number of strings from A. to construct $A*$:

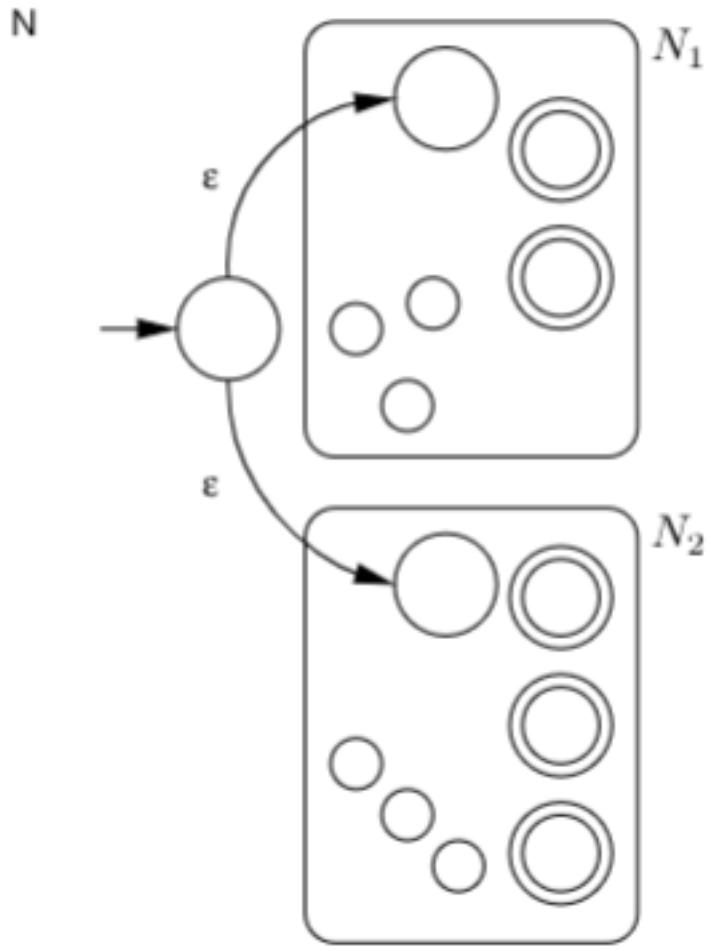

Figure 1: a union of L1 and L2

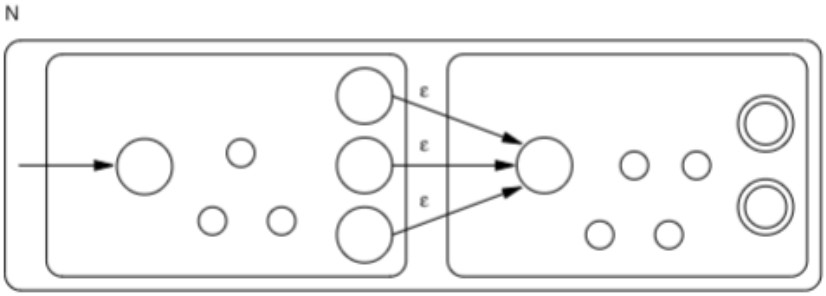

Figure 2: concatenation of A and B

- draw the automaton of A.
- add a new start state and make it an accept state.
- add ϵ from the new start state to the original start state of A
- for each *original* accept state of A, add ϵ transition to the *original* start state of A.

it looks like this:

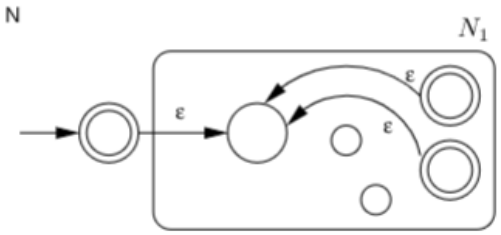

Figure 3: star operation